2.1.8 Divergenz des Elektrischen Feldes

a) Aus $U = \Delta V = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\, d\vec{s}$ (P)

$$= -\int_{\vec{r}_1}^{\vec{r}_2} \left(E_x\, dx + E_y\, dy + E_z\, dz \right)$$

für $x_1 \to x_2$, $y_1 \to y_2$, $z_1 \to z_2$:

$$\Delta V_x = -E_x\, dx \quad \Rightarrow \quad E_x = -\frac{\partial V}{\partial x}$$

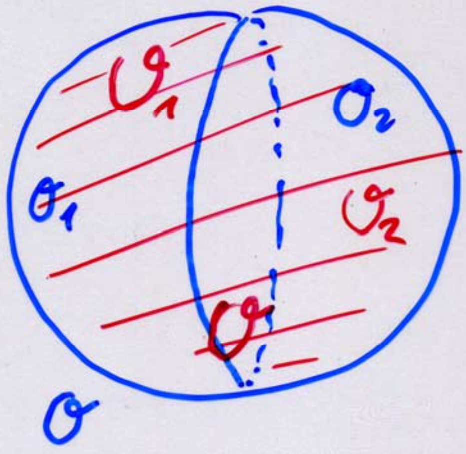
$$\Delta V_y = -E_y\, dy \quad \Rightarrow \quad E_y = -\frac{\partial V}{\partial y}$$

$$\Delta V_z \quad \ldots \quad \Rightarrow \quad E_z = -\frac{\partial V}{\partial z}$$

$$\underline{\vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r})}$$

b) Flußgleichung

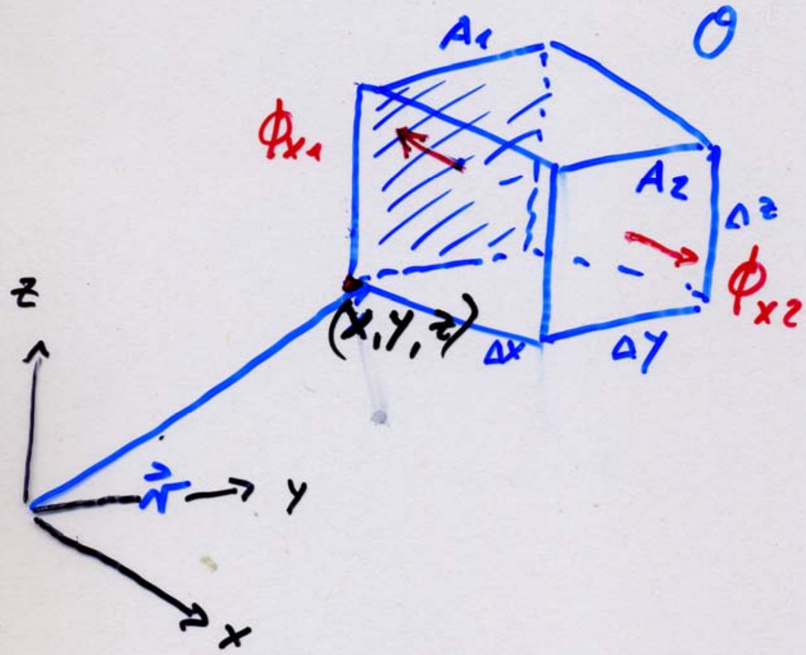
$$\oint_O \vec{E}\, d\vec{A} = \frac{Q}{\varepsilon_0}$$



$$\oint_O \vec{E}\, d\vec{A} = \oint_{O_1} \vec{E}\, d\vec{A} + \oint_{O_2} \vec{E}\, d\vec{A}$$

$$= \sum_i^N \oint_{O_i} \vec{E}\, d\vec{A}$$

$$\rightarrow \int_V dV \lim_{\Delta V \to 0} \frac{1}{\Delta V} \int_{O_{V_i}} \vec{E}\, d\vec{A}$$



Fluß ϕ

$$\phi_{x1} = -\int_{A_1} E_x \, dy\, dz \approx -E_x(x,y,z)\cdot\Delta y\cdot\Delta z$$

$$\phi_{x2} = +\int_{A_2} E_{x+\Delta x}\, dy\, dz \approx E_x(x+\Delta x, y, z)\cdot\Delta y \cdot\Delta z$$

$$\phi_{x1} + \phi_{x2} = \phi_x = \frac{\partial E_x}{\partial x}\,\Delta x\,\Delta y\,\Delta z$$

Analog für ϕ_y, ϕ_z

$$\oint_O \vec{E}\, d\vec{A} = \Delta x\,\Delta y\,\Delta z \cdot \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right)$$

$$= \Delta V \cdot \vec{\nabla}\vec{E}$$

$$\lim_{\Delta V \to 0} \frac{1}{\Delta V} \oint_{O} \vec{E}\, d\vec{A} = \nabla \vec{E}$$

Divergenz

$$\boxed{\oint_{O} \vec{E}\, d\vec{A} = \frac{1}{\varepsilon_0} Q} = \int_{V} \nabla \vec{E}\, dV$$

$$Q = \int \rho\, dV$$

$$= \int_{V} \frac{1}{\varepsilon_0} \rho\, dV$$

$$\Rightarrow \boxed{\nabla \vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}}$$

1. Maxwellgleichung in diff. Form

Auch : mit $\vec{E} = -\nabla V$:

$$-\nabla(\nabla V) = \boxed{\frac{\rho}{\varepsilon_0} = -\Delta V}$$

Poissongleichung

$$\Delta = \vec{\nabla}\vec{\nabla}$$

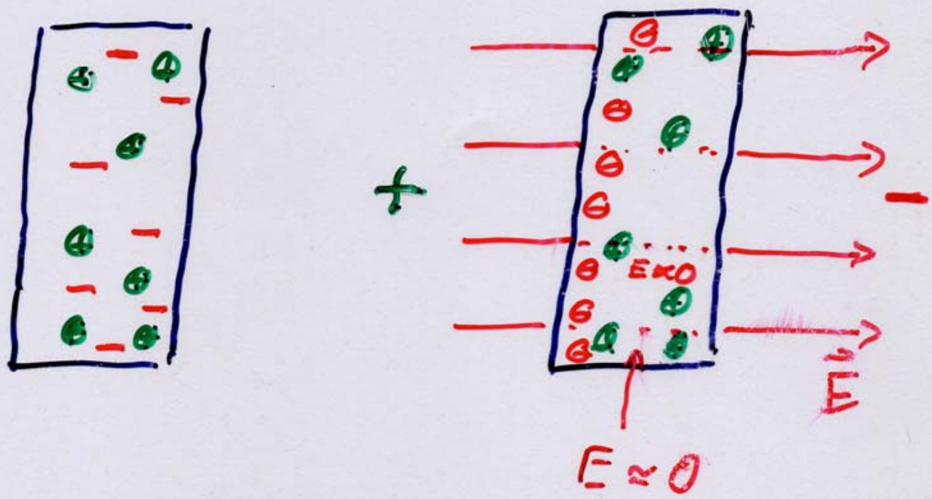
$$= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

Laplace - Operator

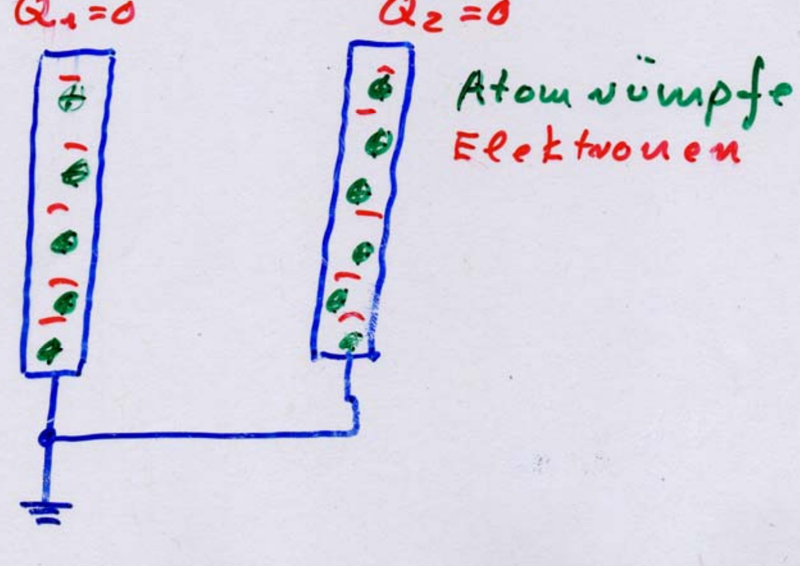
2.2. Leiter und Isolatoren im Elektrischen Feld

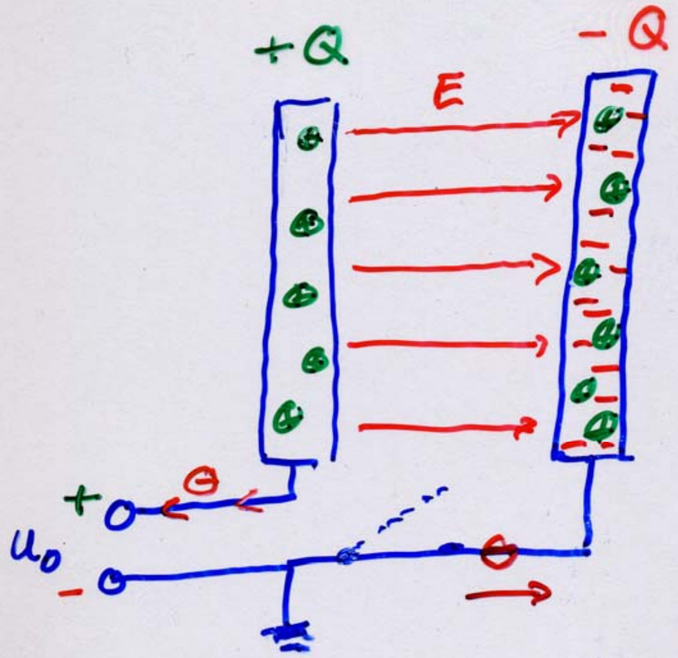
Leiter im Feld: Ladungen werden bewegt, bis Kräfte im Gleichgewicht

zB Metall:



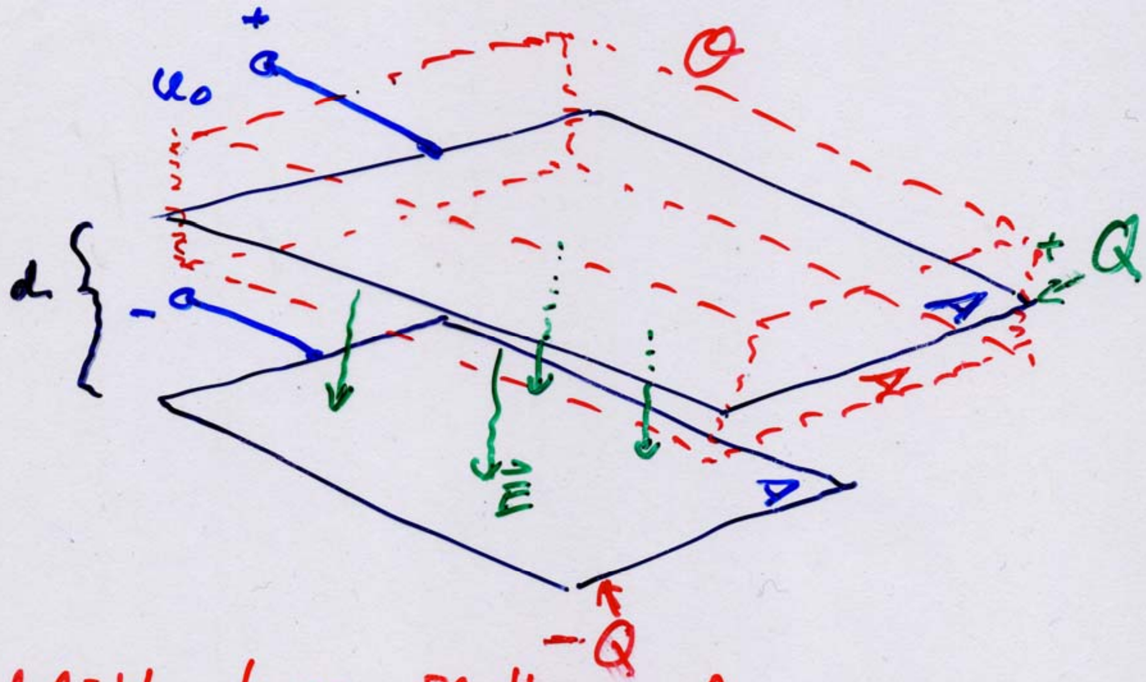
2.2.1 Kondensatoren





Kondensator : speichert Ladungen

Spezialfall Plattenkondensator



O umschließt obere Platte A .

a) <u>El. Feld</u>

Fluß : $\Phi = \int_O \vec E \, d\vec A = \int_A \vec E \, d\vec A + \underbrace{\int_{A_{rest}} \vec E \, d\vec A}_{=0}$

$$\phi = \int_A E\, dA = E \cdot A$$

$$= \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{A \cdot \varepsilon_0}$$

b) Spannung

$$U_0 = \Delta V = \int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\, d\vec{s}$$

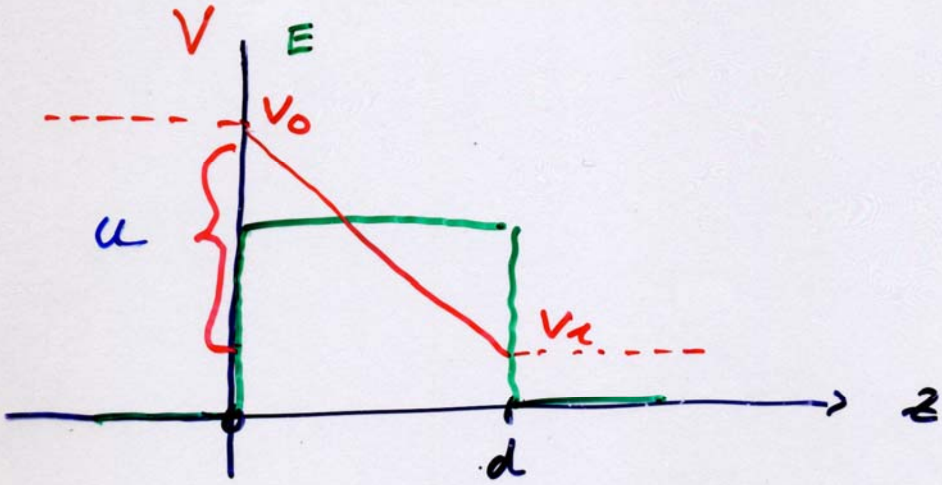
$$= E \cdot d$$

$$= \frac{Q}{\varepsilon_0 A} \cdot d$$

$$\Rightarrow Q = \frac{\varepsilon_0 A}{d} \cdot U_0$$ (Plattenkond.)

$$\boxed{Q = C \cdot U}$$ (Allgemein)

Kapazität



Bs: $A = 100\ \text{cm}^2$, $d = 1\ \text{mm}$

$$\Rightarrow \quad C = 88{,}5 \cdot 10^{-12} \ \underbrace{\frac{C}{V}}_{= F}$$

Farad

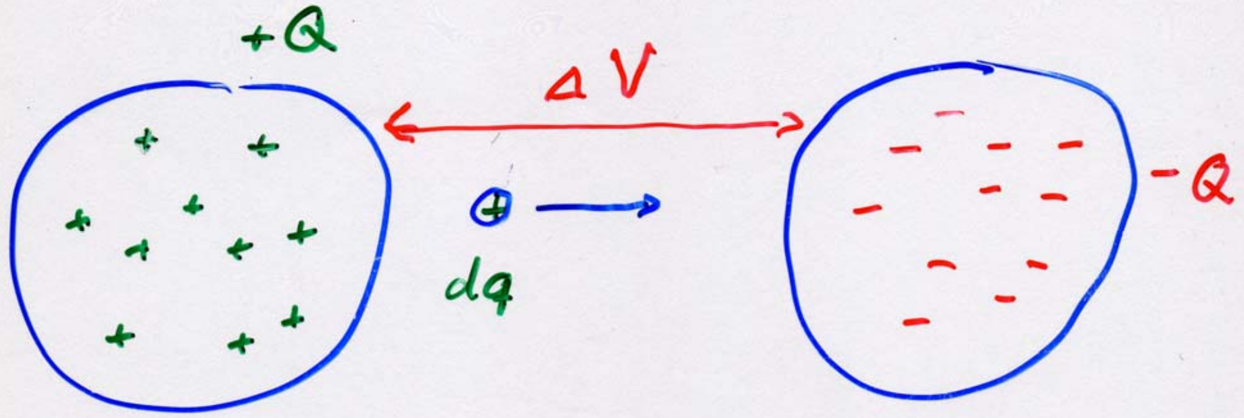
Typische Größen :

$10^{-6}\ F \equiv 1\ \mu F$: Netzteile

$10^{-9}\ F \equiv 1\ nF$: Verstärker (Musik)

$10^{-12}\ F \equiv 1\ pF$: Hochfrequenzschaltungen

2.2.2 Energie des Kondensators



$$dW = \Delta V \cdot dq = \frac{q}{C} \cdot dq$$

$$\Rightarrow W = \frac{1}{C} \int_0^Q q \, dq$$

$$= \frac{Q^2}{2C} = \frac{1}{2} C U^2$$

Gespeicherte Energie

Bsp: Plattenkondensator:

$$W = \frac{1}{2} C U^2$$

$$= \frac{1}{2} \varepsilon_0 A d \frac{U^2}{d^2}$$

$$= \frac{1}{2} \varepsilon_0 V \cdot E^2$$

Energie im Elektrischen Feld

Demo :

a) $C = 100\,\mu F$ (!)

$U = 16\ V$

$\Rightarrow W = \frac{1}{2}\ 10^{-1}\ F \cdot (16\,V)^2$

$= 12\ Ws$

b) $C = 4\,\mu F$

$U = 3{,}3\ kV$

$\Rightarrow W = 4 \cdot 10^{-6} F \cdot (3{,}3\ 10^3\ V)^2$

$\approx 20\ Ws$