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Measurement of the effect of quantum phase slips in a Josephson junction chain

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The interplay between superconductivity and Coulomb interactions has been studied for more than 20 years now[1–13]. In low-dimensional systems, superconductivity degrades in the presence of Coulomb repulsion: interactions tend to suppress fluctuations of charge, thereby increasing fluctuations of phase. This can lead to the occurrence of a superconducting-insulator transition, as has been observed in thin superconducting films[5,6], wires[7] and also in Josephson junction arrays[4,9,11–13]. The last of these are very attractive systems, as they enable a relatively easy control of the relevant energies involved in the competition between superconductivity and Coulomb interactions. Josephson junction chains have been successfully used to create particular electromagnetic environments for the reduction of charge fluctuations[14–16]. Recently, they have attracted interest as they could provide the basis for the realization of a new type of topologically protected qubit[17,18] or for the implementation of a new current standard[19]. Here we present quantitative measurements of quantum phase slips in the ground state of a Josephson junction chain. We tune *in situ* the strength of quantum phase fluctuations and obtain an excellent agreement with the tight-binding model initially proposed by Matveev and colleagues[8].

In superconducting circuits, each electrical element such as an inductor, a capacitor or the Josephson element can add a degree of freedom. In the case of small circuits, by applying Devoret's circuit theory[20], a complete analytical description that takes into account all degrees of freedom can be obtained. However, when the circuits contain an increasing number of elements, as for example Josephson junction chains, even numerical solutions of the problem become difficult to obtain when taking into account all degrees of freedom. Nevertheless, our measurements demonstrate that the ground state of a phase-biased Josephson junction chain (see Fig. 1a) can be described by a single degree of freedom. Although the chain is a multidimensional object, the effect of quantum phase slips can be described by a single variable, m, that counts the number of phase slips in the chain.

We start by giving a short introduction on the low-energy properties of a Josephson junction chain analysed in terms of quantum phase slips[8]. Let us consider the Josephson junction chain shown in Fig. 1a. The chain contains N junctions and is biased with a phase γ. We denote E_J the Josephson energy of a single junction and $E_C = e^2/2C$ its charging energy. Here we consider $E_J \gg E_C$. Let Q_i be the charge on each junction and θ_i the phase difference. In the nearest-neighbour-capacitance limit, the Hamiltonian can be written as:

$$H = \sum_{i=1}^{N} [4E_C(Q_i/2e)^2 + E_J(1-\cos\theta_i)]\ ;\ \sum_{i=1}^{N}\theta_i = \gamma$$

Ignoring the charging energy for the moment, in the classical ground state the phase bias γ is equally distributed on the N junctions: $\theta_i = \gamma/N$, as illustrated by the solid line in Fig. 1b. The resulting Josephson energy hence reads $E_0 = E_J\gamma^2/2N$ and the chain is equivalent to a large inductance. If a phase slip occurs on one of the junctions, say the jth junction, then $\theta_j \to \theta_j + 2\pi$. The constraint $\sum_i \theta_i = \gamma$ would be violated after such a phase-slip event if the phases across all other junctions do not adjust. Therefore, the phase difference θ_i over all other junctions changes a little from γ/N to $(\gamma-2\pi)/N$ to accommodate the bias constraint (see the dashed line in Fig. 1b). A phase slip on a single junction leads to a collective response of all junctions. Consequently, the Josephson energy of the entire chain changes from $E_0 = E_J\gamma^2/2N$ to $E_m = E_J(\gamma-2\pi m)^2/2N$ after m phase slips. The classical ground state energy of the chain consists of shifted parabolae that correspond respectively to a fixed number m of phase slips (see Fig. 1c). For the special values $\gamma = \pi(2m+1)$, the energies E_m and E_{m+1} are degenerate.

Taking now into account the finite charging energy E_C, quantum phase slips can lift the degeneracy at the points $\gamma = \pi(2m+1)$. In the limit of rare phase slips, that is, $E_J \gg E_C$, the hopping element for the quantum phase slip can be approximated by[21,22]: $\nu = 16\sqrt{E_J E_C/\pi}(E_J/2E_C)^{1/4} e^{-\sqrt{8E_J/E_C}}$. As a phase slip can take place on any of the N junctions, the hopping term between the two states $|m\rangle$ and $|m+1\rangle$ is given by $N\nu$. Therefore, using a tight-binding approximation, the total Hamiltonian for the chain is given by:

$$H|m\rangle = E_m|m\rangle - N\nu\,[|m-1\rangle + |m+1\rangle] \tag{1}$$

Figure 1c shows the numerical calculation of the two lowest eigenenergies of the Hamiltonian (1) for three different ratios $E_J/E_C = 20$, 3 and 1.3 in the case of a six-junction chain. Figure 1d shows the corresponding current–phase relation of the chain in the ground state. The chain's supercurrent is obtained by the calculation of the derivative of the ground-state energy E_g: $i_S = (2e/\hbar)(\partial E_g/\partial\gamma)$. For large values of E_J/E_C, quantum phase fluctuations are very small ($\nu \sim 0$) and the current–phase relation has a sawtooth-like dependence with a critical current that is approximatively N/π times smaller than that of a single junction of the chain. We call this regime the 'classical' phase-slip regime. When quantum phase fluctuations increase, that is, E_J/E_C decreases, the current–phase relation becomes sinusoidal and the critical current becomes exponentially suppressed with N and E_J/E_C (ref. 8).

To measure the effect of quantum phase slips on the ground state of a Josephson junction chain, we have studied a chain of six junctions. Our measurement set-up and the junction parameters are presented in Fig. 2 and Table 1. Each junction in the chain is realized by a superconducting quantum interference device (SQUID) to enable tunable Josephson coupling E_J. In this way we

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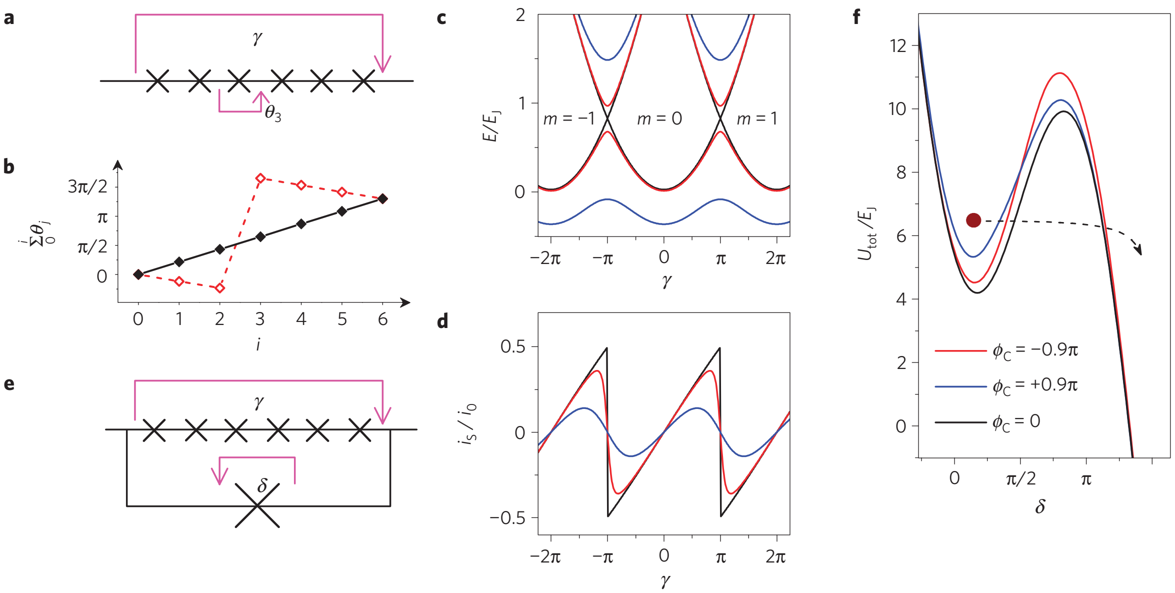

Figure 1 | Graphic representations describing the effect of phase slips in a six-junction chain, the resulting chain's energy and supercurrent and the measurement principle. a, Schematic picture of the phase-biased Josephson junction chain. **b**, Representation of a phase slip in the chain. The filled diamonds show the initial configuration. The open diamonds show the phase configuration after a 2π flip of the phase on the third junction θ_3. **c**, Energy levels of a Josephson junction chain with $N=6$ as a function of bias phase γ for different ratios E_J/E_C. For $E_J/E_C=20$ (black lines) no splitting is visible at the crossing points. For $E_J/E_C=3$ (red lines) a gap emerges that increases rapidly with decreasing E_J/E_C. The blue lines show the energy levels for $E_J/E_C=1.3$. For each E_J/E_C, the two lowest-lying states have been calculated by numerical diagonalization of the Hamiltonian (1). **d**, Current–phase relation for the ground state $E_g(\gamma)$ for the same E_J/E_C ratios as in **c**. The supercurrent is calculated from the derivative of the energy band: $i_S=(2e/\hbar)(\partial E_g/\partial\gamma)$. The chain current is reported in units of the critical current of a single chain junction $i_0=(2e/\hbar)E_J$. **e**, Schematic picture of the chain shunted by the read-out junction. **f**, Escape potential for the Josephson junction chain with $E_J/E_C=3$ in parallel with the read-out junction for three different flux biases ϕ_C in the read-out loop (see Fig. 2). The ground state of the chain clearly modifies the escape potential of the read-out junction.

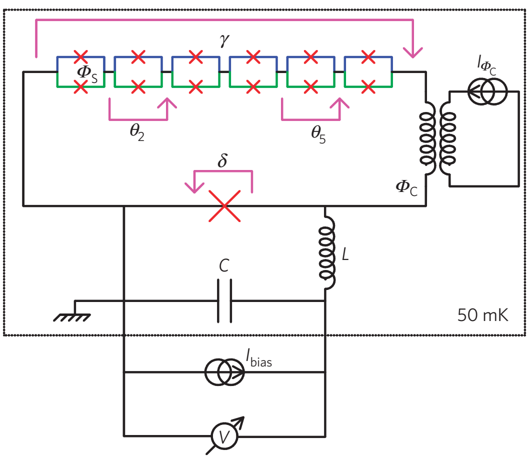

Figure 2 | Measurement circuit. The six-SQUID chain is inserted in a superconducting loop. The flux Φ_C created by on-chip coils controls the phase difference γ over the chain. The flux Φ_S through the SQUIDs can be controlled independently by a second coil. We denote the phase difference over the read-out junction δ.

can tune *in situ* the E_J/E_C ratio by applying a uniform magnetic flux Φ_S through all SQUIDs, and consequently we can control the strength of quantum phase fluctuations. For our measurements we placed this chain in a closed superconducting loop, threaded by the flux Φ_C, containing an extra shunt Josephson junction that is used for the read-out of the chain state. The flux Φ_C enables the control of the bias phase $\gamma=\Phi_C-\delta$ over the chain.

We have measured the switching current of the entire Josephson junction circuit containing both the chain and the read-out junction. The switching current was determined from the switching

Table 1 | Parameters of the sample: size, capacitance, normal-state resistance and critical current of the read-out junction and a single SQUID of the chain.

Read-out junction	SQUID at $\phi_S=0$
$S^{RO}=(121\pm5)\times10^3\ \mathrm{nm}^2$	$S^{SQ}=(30\pm2)\times10^3\ \mathrm{nm}^2$
$C^{RO}=5.8\pm0.2$ fF	$C=1.4\pm0.1$ fF
${R_N}^{RO}=968\pm5\ \Omega$	${R_N}^{SQ}=3{,}800\pm450\ \Omega$
${I_C}^{RO}=330\pm2$ nA	${I_C}^{SQ}=83\pm9$ nA

The critical-current variance for the junctions in the chain is estimated to be smaller than 4%.

probability at 50%. The switching probability as a function of bias current I_{bias} has a width of $\approx$20 nA. We apply typically 10,000 bias-current pulses of amplitude I_{bias} and measure the switching probability as the ratio between the number of switching events and the total number of pulses. The current pulses have a rise time of 8 µs and a total duration of 20 µs. The results of the switching-current measurements as a function of flux Φ_C are shown in Fig. 3. From these switching-current measurements we deduce the effect of quantum phase slips on the ground state of the chain.

The measured switching current corresponds to the escape process out of the total potential energy U_{tot} containing the contributions of the read-out junction and the chain:

$$U_{tot}(\delta,\Phi_C)=E_J^{RO}\cos(\delta)+E_g(\Phi_C-\delta)-\frac{\hbar}{2e}I_{bias}\delta$$

Here E_g is the ground state of the six-SQUID chain calculated by solving the Hamiltonian (1). As $E_J^{RO}\gg E_g$ the main component in U_{tot} is the potential of the current-biased read-out junction $E_J^{RO}\cos(\delta)-(\hbar/2e)I_{bias}\delta$. Figure 1f shows the escape potential at constant bias current for three different flux values ϕ_C

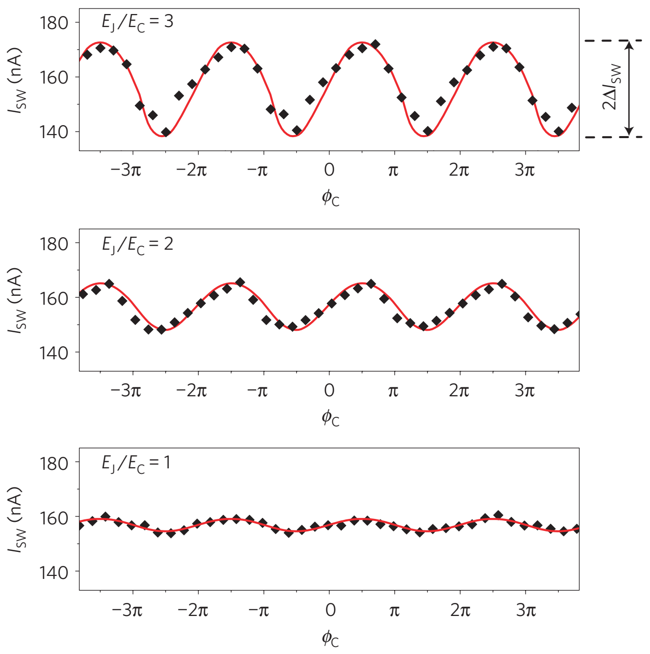

Figure 3 | Measured switching current (black diamonds) as a function of ϕ_C over the chain for three different E_J/E_C ratios. The measurement noise for each point is about 0.2 nA. The red lines represent theoretical calculations for the switching current using equations (3) and (2).

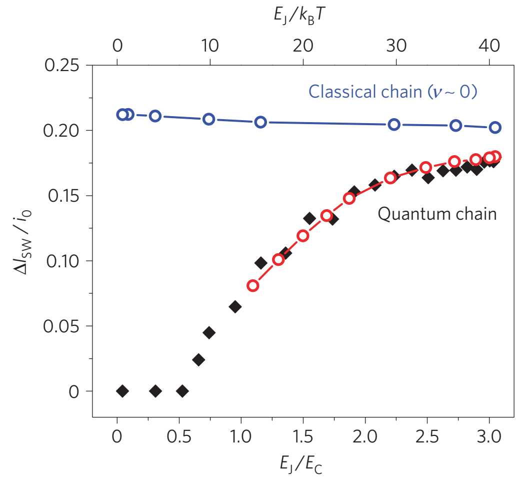

Figure 4 | Comparison between the measured and the calculated switching-current amplitude as a function of the E_J/E_C ratio. Black diamonds: measured; red open circles: calculated. Note that the switching-current amplitude is divided by the flux-dependent critical current of a single SQUID i_0, to reveal the effect of quantum-phase fluctuations. The top curve (blue open circles) shows the theoretical calculation of the switching-current amplitude in the absence of quantum phase fluctuations. The lines are guides for the eye.

corresponding to three different biasing phases γ over the chain. Let us point out that the position of the minimum of the potential U_{tot} is in good approximation independent of the value of the flux ϕ_C. Therefore, the bias phase difference γ over the chain depends only on the flux ϕ_C. As a consequence, the ϕ_C dependence of the measured switching current results from the γ dependence of the chain's ground state.

The escape from the potential U_{tot} occurs by means of macroscopic quantum tunnelling (MQT). The MQT rate for an arbitrary potential can be calculated in the limit of weak tunnelling using the dilute instanton-gas approximation[23]. Within this model, the escape rate Γ out of the washboard potential $U_{tot}(\gamma)$ reads[24]:

$$\Gamma = A\exp[-B]$$

where A and B are given by:

$$A = \sqrt{\frac{\hbar\omega_0{}^3}{8\pi\hbar}}\sigma e^I \quad \text{with} \quad I = 2\int_0^\sigma \sqrt{\frac{\hbar^2 U_{tot}(x)}{4E_C{}^{RO}}}\mathrm{d}x$$
$$B = \int_0^\sigma \left[\sqrt{\frac{\hbar^2\omega_0{}^2}{16E_C{}^{RO}U_{tot}(x)}} - \frac{1}{x}\right]\mathrm{d}x \qquad (2)$$

We have denoted by σ the width of the barrier and by x the phase coordinate measured from the minimum of the washboard potential. The plasma frequency is $\omega_0 = \sqrt{8E_C{}^{RO}U_{tot}{}''(0)/\hbar}$, where $E_C{}^{RO}$ is the charging energy of the read-out junction.

Knowing the escape rate Γ, we can calculate the switching probability:

$$P(I_{bias}) = 1 - \exp[-\Gamma(I_{bias})\Delta t] \qquad (3)$$

The results of numerical calculations and the experimental data are shown in Fig. 3. The theory fits very well both in amplitude and shape the oscillations of the measured switching current. Let us point out that we have used the nominal values for E_J and E_C calculated from the characteristics of the sample indicated in Table 1. The normal-state resistance for a single chain junction has been deduced from the measured normal-state resistance of the read-out junction by considering the size ratio between the two. We evaluate the precision of the determination of E_J and E_C to be in the range of $\pm10\%$. This error bar on E_J and E_C yields an uncertainty of $\pm15\%$ for the phase-slip amplitude $N\nu$. The eventual presence of junction inhomogeneity or an important effect of background charges would imply a significantly larger decrease of the phase-slip amplitude[8]. The good agreement between theory and experiment confirms the homogeneity of our junctions. It excludes a significant contribution of background charges in the overall shape of the switching curve and demonstrates the collective nature of the phase-slip events.

From the measurements in Fig. 3, we define the switching-current amplitude ΔI_{SW} as half of the peak-to-peak variation of the switching current with the flux Φ_C. Figure 4 shows the measured ΔI_{SW} and the corresponding theoretical calculations as a function of E_J/E_C. For each measurement, E_J has been calculated using the flux dependence of the SQUID's Josephson coupling: $E_J(\Phi_S) = (\hbar/2e)i_0(\Phi_S)$ with $i_0(\Phi_S) = I_C{}^{SQ}\cos(\pi\Phi_S/\Phi_0)$. To distinguish between the suppression of the switching current that is due to quantum phase fluctuations and the one that is simply due to the well-known cancellation of the SQUID's critical current as a function of flux, we plot the switching-current amplitude divided by the critical current of a single SQUID i_0. We see that the measured switching-current amplitude follows very well the predicted theoretical suppression of the switching-current oscillations in the presence of quantum phase fluctuations. From our measurements we can also deduce the strength of the quantum phase-slip amplitude. With decreasing E_J/E_C ratio from 3 to 1 the quantum phase-slip amplitude increases from 0.8 to 2.7 GHz. In addition, in Fig. 4 we have plotted for comparison the calculation for the switching-current amplitude in the case when quantum phase fluctuations would be negligibly small: $\nu \sim 0$. As expected, we get a practically flat dependence on E_J/E_C.

Further on, the upper x axis of Fig. 4 shows the ratio E_J/k_BT of the Josephson energy with respect to the thermal energy at $T = 50\,\mathrm{mK}$. As $E_J \gg k_B T$, thermal fluctuations are excluded to explain the suppression of the switching current with decreasing E_J/E_C. Further measurements (not shown here) reveal a constant switching-current amplitude and width of the switching distribution up to a temperature of $T = 100\,\mathrm{mK}$.

We present a detailed experimental characterization of the effect of quantum phase slips on the ground state of a Josephson junction chain. These phase slips are the result of fluctuations induced by the finite charging energy of each Josephson junction in the chain. The experimental results can be fitted in very good agreement by considering a simple tight-binding model for the phase slips[8]. Our measurements also show that a Josephson junction chain under phase-bias constraint can behave in a collective way very similar to a single macroscopic quantum object.

These results open the way for the use of quantum phase slips in Josephson junction networks for the implementation of a new current standard, the observation of Bloch oscillations[19], the fabrication of topologically protected qubits[25] and the design of new superconducting circuit elements.

Methods

The circuit was fabricated on a Si/SiO_2 substrate and the $Al/AlO_x/Al$ junctions were obtained using standard shadow evaporation techniques. The aluminium oxide was obtained by natural oxidation in a controlled O_2 atmosphere. Room-temperature measurements on an ensemble of ~100 junctions revealed a variance of 4% of the normal-state resistance of the junctions.

The sample was mounted in a closed copper block that was thermally connected to the cold plate of a dilution refrigerator at 50 mK. All lines were strongly filtered by low-pass filters at the cryostat entrance and by thermocoaxial cables and π filters at low temperatures.

The switching current I_{SW} of the circuit is obtained by carrying out the following sequence. We use a series of M current steps of equal amplitude I_{bias} to bias the junction. We count the number of transitions to the voltage state M_{SW} and thus obtain the value of the switching probability $P_{SW} = M_{SW}/M$ corresponding to the applied I_{bias}. By sweeping the I_{bias} amplitude and repeating the above sequence, we measure a complete switching histogram, P_{SW} versus I_{bias}. The $P_{SW} = 50\%$ bias current is called the switching current of the circuit, I_{SW}.

The principle of the read-out scheme was first implemented by Vion *et al.*[26] and has also been used for the measurement of the ground state of superconducting atomic contacts[27]. The choice of the read-out junction critical current I_C^{RO} for an optimal measurement of i_S is not straightforward. On the one hand one would like to have $I_C^{RO} \gg i_S$, but on the other hand the width of the switching histograms w increases with I_C^{RO} and so do the statistical fluctuations resulting from finite ensemble size $w/\sqrt{M}$. For reasonable measuring timescales, the number of current steps M is limited to values of about 10^4. If we want to measure supercurrents for the SQUID chain in the range of 1 nA, I_C^{RO} needs to be in the range of 100 nA. We have used a read-out junction with a critical current $I_C^{RO} = 330\,\mathrm{nA}$ that offers a good trade-off.

In our MQT analysis we neglect the effect of dissipation on the escape rate. Small dissipation can add a pre-factor in front of the exponential in the switching probability formula (3). However, this factor is independent of Φ_C, so it will change only the offset value of I_{SW} in Fig. 3, but not the shape nor the amplitude of the I_{SW} oscillations.

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Author contributions

I.M.P. fabricated the sample and carried out the experiments. I.M.P., B.P., O.B. and W.G. designed the experiment, analysed the data and wrote the paper. I.M.P. and I.P. carried out the numerical calculations. Z.P. and F.L. contributed to the carrying out of experiments and sample fabrication. W.G. supervised the project. All authors discussed the results and commented on the manuscript at all stages.

Additional information

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