Optical frequency comb and precision spectroscopy

- Fundamental constants
- Are fundamental constants constant?
- Precision spectroscopy
- Frequency comb
- Drift of fine structure constant
Standard Model of FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the unified theory of weak and electromagnetic interactions (electroweak). Gravity is included on this chart because it is one of the fundamental interactions even though not part of the "Standard Model."

**FERMIONS**  
**Leptons** spin = 1/2  
- $\nu_e$ electron neutrino  
- $e^-$ electron  
- $\nu_\mu$ muon neutrino  
- $\mu^-$ muon  
- $\nu_\tau$ tau neutrino  
- $\tau^-$ tau

**Quarks** spin = 1/2  
- $u$ up  
- $d$ down  
- $c$ charm  
- $s$ strange  
- $t$ top  
- $b$ bottom  
- $\bar{u}$ antiquark of up  
- $\bar{d}$ antiquark of down  
- $\bar{c}$ antiquark of charm  
- $\bar{s}$ antiquark of strange  
- $\bar{t}$ antiquark of top  
- $\bar{b}$ antiquark of bottom

**Spin** is the intrinsic angular momentum of particles. Spin is given in units of $\hbar$, which is the quantum unit of angular momentum, where $\hbar = \sqrt{\frac{\hbar}{2\pi}} = 6.58 \times 10^{-34} \text{ kg m}^2 \text{s}^{-1}$.

**Electric charge** is given in units of the proton's charge. In SI units the electric charge of the proton is $1.60 \times 10^{-19} \text{ Coulomb}$.

**Energy** of an electron is the electronvolt (eV), the energy gained by one electron in traversing a potential difference of one volt. Masses are given in GeV/c^2 (1 GeV = 10^9 eV), where 1 eV = 1.60 \times 10^{-19} \text{ Joule}. The mass of the proton is 0.938 GeV/c^2 = 1.67 \times 10^{-27} \text{ kg}.

**Electromagnetic Force**  
- Charged particles interact via electrostatic forces. Positively charged particles attract negatively charged particles, and negatively charged particles attract positively charged particles. Electric forces are strongest at short distances and diminish with the square of the distance.

**Weak Interactions**  
- Neutrino interactions are mediated by the weak force. The weak force is responsible for processes such as beta decay, weak interactions, and the emission of neutrinos.

**Strong Interactions**  
- Quark interactions are mediated by the strong force. The strong force binds quarks together in hadrons (protons and neutrons).

**Gravitational Force**  
- Gravitational force is a force that acts between any two objects with mass. The force is inversely proportional to the square of the distance between the objects.

**Properties of the Interactions**

- **Baryons**  
  - qqq  
  - qqq  

- **Mesons**  
  - qq  

**Matter and Antimatter**  
- For every particle type there is a corresponding antiparticle type, denoted by a bar over the symbol (e.g., $\bar{e}$ = electron antineutrino).

**Figures**  
- The Standard Model is a theoretical framework that describes the fundamental forces and particles of nature. It is based on the principles of quantum mechanics and special relativity.

**The Particle Adventure**  
- Visit the award-winning website, "The Particle Adventure" at http://ParticleAdventure.org

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http://CPEPweb.org
Fundamental constants

Universal constants

\( \varepsilon_0, \mu_0, c, G, \hbar, \)
\( e, m_e, m_p, k_B \)

Physical-chemical constants

\( \sigma, ... \)

Electromagnetic constants

\( \mu_B, \mu_n, ... \)

Frequently asked constants

\( c, e, m_e, m_p, \hbar \)

Dirac’s assumption about large numbers

Dirac’s assumption about large numbers

\[ E_G = G \frac{m_e m_p}{r^2} \]
\[ E_e = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} \]
\[ \frac{E_e}{E_G} \sim 10^{39} \]

\[ T_{\text{Universe}} \sim 10^{10} \text{ y} \rightarrow \frac{T_{\text{Universe}}}{e^2 / mc^3} \sim 10^{40} \]

This suggests that the above-mentioned large numbers are to be regarded, not as constants, but as simple functions of our present epoch, expressed in atomic units.

P. A.M. Dirac, Nature 139, 323 (1937)
Einstein equivalence principle

(1) Weak (Galilean) equivalence principle:

“Mass (measured with balance) and weight (measured with scale)” are locally in identical ration for all bodies”

(2) Einstein equivalence principle:

“The outcome of any local non-gravitational experiment is independent of the velocity of the laboratory in space-time”

(3) Local time and position invariance:

“The outcome of any local non-gravitational experiment is independent of time and its position in space-time”
Einstein equivalence principle

In all theories of gravity, including general relativity,

any drift of non-gravitational constants

IS FORBIDDEN!

\[ \frac{\partial \alpha}{\partial t} \equiv 0 \]
Theories towards unification of quantum mechanics and gravity allow for, or even predict violation of EEP.

\[ \frac{\partial \alpha}{\partial t} \neq 0 \] is allowed
Possible origins for the drift

General Physics

Fundamental Constants

Boundary Conditions

Inflation

Big Bang

Dark matter

Expansion

Extra-Dimensions
Search for Drift

\[ \Theta(t_1) \] \quad \Theta(t_2) \quad \Theta \quad \text{- stable value (dimensionless)}

\[ \Theta(\alpha_1, \alpha_2, \ldots) \quad \text{depends on a set of fundamental parameters} \quad \text{e.g. frequency ratio, ...} \]

\[ \frac{\Theta(t_2) - \Theta(t_1)}{t_2 - t_1} \quad \text{Model} \quad \frac{\partial \alpha_i}{\partial t} \]
Sensitivity to the DRIFT is much higher!
Experiments to find the drift of alpha:

Keck versus VLT

www.keckobservatory.org  www.eso.org
Atomic spectra and astrophysics

<table>
<thead>
<tr>
<th>Transition</th>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical</td>
<td>Ry</td>
</tr>
<tr>
<td>Fine structure</td>
<td>$\alpha^2$ Ry</td>
</tr>
<tr>
<td>Hyperfine structure</td>
<td>$\mu_{\text{nucl}} / \mu_B \alpha^2$ Ry</td>
</tr>
</tbody>
</table>

\[ z = \frac{v_{\text{emission}}}{v_{\text{today}}} - 1 \]

FS/Optical $\sim \alpha^2$ and does not depend on red shift $z$

Many multiplet method

\[ f = f_{NR} \cdot F_{rel}(Z\alpha) \]

\( F_{rel}(Z\alpha) \) depends only on \( \alpha \)

\[
\frac{\partial}{\partial t} \ln \frac{f_1}{f_2} = \frac{\partial \ln \alpha}{\partial t} \times \frac{\partial}{\partial \ln \alpha} \ln \frac{F_{rel}(1)}{F_{rel}(2)}
\]

measurable value

to be determined

calculated from basic principles
Many multiplet method

\[
\Delta \alpha / \alpha = (0.1 \pm 1.7) \cdot 10^{-6}
\]


Keck / HIRES (72 systems):
\[
\Delta \alpha / \alpha = (-0.57 \pm 0.1) \cdot 10^{-5}
\]
\[
\dot{\alpha} / \alpha \approx 10^{-15} \text{ yr}^{-1}
\]

Natural fission $^{235}\text{U}/^{238}\text{U}$ reactor
2·$10^9$ years ago

\[ n + ^{149}_{62}\text{Sm} \rightarrow ^{150}_{62}\text{Sm} + \gamma \]
resonance width 0.1 eV


Abundance ratio:

\[ \frac{^{149}_{62}\text{Sm}}{^{147}_{62}\text{Sm}} \]

0.02 (Oklo)

0.9 (typical)

\[ \Delta \alpha/\alpha = (-0.36\pm1.44) \cdot 10^{-8} \]


Long “observation time”: systematic effects!

Oklo: Location
Comparison of methods

Astrophysical (Many multiplet)

$\frac{\Delta \alpha}{\alpha} \sim 10^{-6}$
$\Delta T \sim 10^{10}$ Years

J.K. Webb et al., PRL (2001)

Geological (Oklo)

$\frac{\Delta \alpha}{\alpha} \sim 10^{-8}$
$\Delta T \sim 10^{8}$ Years


Experimental Lab

$\frac{\Delta \alpha}{\alpha} < 1 \times 10^{-16}$
$\Delta T \sim 10$ Years

Big Bang

Time

14 GYear
Experiments in the lab

- High sensitivity to drift (< $10^{-16}$)
- Short time interval (~10 years)
- Reproduction of results, many systems to study
- Simple analysis
- Weak dependence on the model and systematic errors
Frequency metrology

HFS in $^{133}$Cs (second in SI)

Narrow atomic resonance
Atomic clock comparison

\[ E_{HFS}(Cs) \propto \alpha^2 \text{Ry} \frac{g_{Cs} \mu_N}{\mu_B} F_{rel}(Cs) \]

\[ E_{opt} \propto \text{Ry} \ F'_{rel}(Z\alpha) \]

\[ \frac{\partial}{\partial t} \ln \frac{f_{Cs}}{f_{opt}} = \frac{\partial}{\partial t} \ln \left( \alpha^2 \ g_{Cs} \ \frac{\mu_N}{\mu_B} \right) + \left[ \frac{\partial}{\partial \ln \alpha} \ln \frac{F_{rel}(Cs)}{F'_{rel}(Z\alpha)} \right] \frac{\partial}{\partial t} \ln \alpha \]

\[ \frac{\partial}{\partial t} \ln \frac{f_{Cs}}{f_{H}} \approx \frac{\partial}{\partial t} \ln \left( g_{Cs} \ \frac{\mu_N}{\mu_B} \alpha^2 + 0.8 - 0 \right) \]
## Sensitivity to $\alpha$ drift

<table>
<thead>
<tr>
<th>Z</th>
<th>Atom</th>
<th>Frequency [Hz]</th>
<th>frac. uncert.</th>
<th>Sensitivity to $\alpha$ drift</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H</td>
<td>2 466 061 413 187 103(46)</td>
<td>$2 \cdot 10^{-14}$</td>
<td>0.00</td>
</tr>
<tr>
<td>20</td>
<td>Ca</td>
<td>455 986 240 494 158(26)</td>
<td>$6 \cdot 10^{-14}$</td>
<td>0.03</td>
</tr>
<tr>
<td>49</td>
<td>In$^+$</td>
<td>1 267 402 452 899 920(230)</td>
<td>$18 \cdot 10^{-14}$</td>
<td>0.21</td>
</tr>
<tr>
<td>70</td>
<td>Yb$^+$</td>
<td>688 358 979 309 312(6)</td>
<td>$0.9 \cdot 10^{-14}$</td>
<td>1.03</td>
</tr>
<tr>
<td>80</td>
<td>Hg$^+$</td>
<td>1 064 721 609 899 143(10)</td>
<td>$0.9 \cdot 10^{-14}$</td>
<td>-3.18</td>
</tr>
</tbody>
</table>

- H, Ca, In$^+$ — "anchor" transitions
- Hg$^+$, Yb$^+$ — sensitive transitions

Energy levels of Atomic Hydrogen

\[ E \text{ [eV]} \]

-3.4 \hspace{1cm} n = 2

-13.6 \hspace{1cm} n = 1

Bohr
1S-2S transition in Atomic Hydrogen

Frequency of the hyperfine centroid of the 1S-2S transition in atomic hydrogen:

\[ f = 2\,466\,061\,413\,187\,103(46) \text{ Hz} \]


Natural linewidth: 1.3 Hz
How to compare transitions of different wavelengths?

HFS in $^{133}$Cs (second in SI)

Optical frequency comb

Laser

Resonator

Narrow atomic resonance
Hydrogen spectroscopy setup

**Laser system**

- 486 nm
- SHG
- AOM
- EOM
- fiber
- dye laser 486 nm
- reference cavity
- servo loop

**Spectroscopy**

- 243 nm
- cryostat with cold nozzle
- H₂ gas discharge
- cryopump
- Lα-detector

**Frequency Measurement**

- 486 nm
- Beat
- frequency comb (referenced to Cs fountain)
- Time resolved photon counting

**Frequency Measurement**

- I
- Beat
- f_r
- f_ce
- x2
Frequency metrology with optical frequency comb

www.mpq.mpg.de
Ultra-short laser pulses

- Consider the light source consisting of many frequency modes
- Calculate the temporal evolution of the field
- The light source emits the short pulses
- The pulse length inversely depend on mode number

\[ E(t) = \sum_{n=0}^{N-1} e^{i(\omega_0 + n\Delta\omega)t} = e^{i\omega_0 t} \sum_{n=0}^{N-1} e^{in\Delta\omega t} = \frac{1 - e^{iN\Delta\omega t}}{1 - e^{i\Delta\omega t}} e^{i\omega_0 t} \]

\[ I(t) \propto |E(t)|^2 = \frac{\sin^2 N\Delta\omega t / 2}{\sin^2 \Delta\omega t / 2} \]

\[ \tau_p \approx \frac{2\pi}{N\Delta\omega} \]

Image: www.lincoln.edu
Formation of the frequency comb: 1 fo
$0.9 \text{ fo} + 1.1 \text{ fo}$
0.8-1.2 (0.05 step) $f_0$
Laser with passive mode synchronisation

\[ \omega_n = n\omega_r + \omega_{CE} \quad \text{at} \quad \omega_{CE} < \omega_r \]

Extending the frequency comb


www.ph.surrey.ac.uk
Self-phase modulation

\[ E(t) = A(t)e^{i\omega_0 t} \]

\[ I(t) = |E(t)|^2 \]

\[ \varphi_{NL} = -n_2 I(t)\omega_0 L / c \]

\[ E(t) \rightarrow E(t)e^{i\varphi_{NL}} \]

- The pulse acquires additional phase shift due to Kerr effect
- Different part of the pulse travel at different speed → Chirped pulse
- The envelope \( A(t) \) remains unchanged
Figure 3 The principle of the single-laser optical synthesizer. A mode with the mode number $n$ at the red wing of the comb and whose frequency is given according to equation (2) by $\omega_n = n\omega_r + \omega_0$ is frequency doubled in a nonlinear crystal. If the frequency comb covers a full optical octave, a mode with the number $2n$ should oscillate simultaneously at $\omega_{2n} = 2n\omega_r + \omega_0$. The beat note between the frequency-doubled mode and the mode at $2n$ yields the offset frequency $2(n\omega_r + \omega_0) - (2n\omega_r + \omega_0) = \omega_0$.
Single-laser frequency synthesizer
Single-laser frequency synthesizer

Radiofrequency
or
Optical frequency

\( I(\omega) \)
\[ \omega \]

Radiofrequency
or
Optical frequency
Single-laser frequency synthesizer

Lock to Cs clocks

\[ \omega_n = n\omega_r + \omega_{CE} \]

Each mode can be used for optical measurement

Million stabilized lasers in one beam!
R. Glauber, Th. Udem, T.W. Hänsch
Frequency metrology with optical frequency comb

**Laser system**
- BBO
- SHG
- 486 nm
- 243 nm
- AOM
- EOM
- Reference cavity
- Dye laser 486 nm
- Servo loop

**Spectroscopy**
- Cryostat with cold nozzle
- H₂ gas discharge
- Cryopump
- Lα-detector
- Beat

**Frequency Measurement**
- 486 nm
- Beat
- Frequency comb (referenced to Cs fountain)
- Time resolved photon counting
- 486 nm
P. B. acknowledges a great help and many good advices from Prof. Dr. Nikolay Kolachevsky (in front) during preparation of this lecture
Laser characterisation

Reference Cavity Characterization

Laser linewidth measurements

Two-cavities beat measurement + 1S-2S broadening measurement:

60 Hz @ 486 nm
Hydrogen spectrometer

- 243 nm
- H\(_2\) gas discharge
- H cold nozzle
- vacuum chamber
- chopper
- LHe-flowthrough cryostat
- \(L_\alpha\) detector
- \(\Delta\tau\) [s]
- detuning [kHz]
- signal [cps]
- \(v < 80\) m/s

FWHM = 1060 Hz
Frequency measurement

\[ f_{\text{laser}} = f_{\text{beat}} + f_{ce} + N \cdot f_r \]

486 nm laser

770 664

Phase lock

Primary Reference
Cs$^{133}$ portable atomic fountain clock

**Diagram:**
- **Lasers**:
  - Two 9 GHz lasers providing optical molasses.
- **RF Cavity**:
  - A 1.3 Hz FWHM RF cavity for interaction.
- **Vacuum Chamber**:
  - Enclosed within magnetic shieldings.
- **Interaction Region**:
  - Points for control and detection.
- **Detuning**:
  - Range from -40 to 40 Hz.
  - Transition probability graph.
  - Lock point at the origin.

**Textual Content:**
- BNM-SYRTE, Observatoire de Paris
- Detuning [Hz]
Comparison of 1999 and 2003 H-data

\[ f_{(2003)} - f_{(1999)} = (-29 \pm 57) \text{ Hz} \]

\( (1S, F = 1, m_F = \pm 1) \leftrightarrow (2S, F' = 1, m'_F = \pm 1) \)
Hydrogen measurements and $\alpha$-drift

$$\frac{\partial}{\partial t} \ln \frac{f_{\text{Cs}}}{f_{\text{H}}} \approx \frac{\partial}{\partial t} \ln \left( g_{\text{Cs}} \frac{\mu_N}{\mu_B} \alpha^{2.8} \right) = (3.2 \pm 6.4) \cdot 10^{-15} \text{ yr}^{-1}$$

$$x \equiv \frac{\partial}{\partial t} \ln (\alpha) \quad \text{rel. } \alpha \text{ drift}$$

$$y \equiv \frac{\partial}{\partial t} \ln \left( g_{\text{Cs}} \frac{\mu_N}{\mu_B} \right) \quad \text{magnetic moment drift}$$

2004:

$$y + 2.8x = (3.2 \pm 6.4) \cdot 10^{-15} \text{ yr}^{-1}$$

Combination of experimental data

MPQ + NIST + PTB

Increase of sensitivity by one order of magnitude

2007:

$$\frac{\dot{\alpha}}{\alpha} = (-3 \pm 3.5) \cdot 10^{-16} \, Yr^{-1}$$

Comparison of Hg$^+$ and Al$^+$ clocks

2008:

$$\frac{\dot{\alpha}}{\alpha} = (-5.3 \pm 7.9) \cdot 10^{-17} \text{ Yr}^{-1}$$

T. Rosenband et al., SCIENCE 319, 1808 (2008)
Optical frequency comb for astrophysics

$z = \frac{V_{\text{emission}}}{V_{\text{today}}} - 1$

- Allan Sandage in 1962 has predicted a drift in the red shift of cosmological objects
- Difficult to detect if observation time is less than 10 Mio. Yr.
- Present day challenge: to measure radial velocity change by 1 cm/sec in 20 Year observation course
- Use optical frequency comb for spectrometer calibration

S. Lopez, SCIENCE 321, 1301 (2008)
Image: HUBBLE
Wavelength calibration with frequency comb

ATM – atmospheric absorption
Fraunhofer absorption lines

reached sensitivity: \( \sim 1 \text{ m/sec} \)